QUANTISATION OF GLOBAL ISOSPIN IN THE SKYRME CRYSTAL

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ABSTRACT: A quantisation of chiral symmetry within the Skyrme crystal is carried out. The definition of global isospin in the crystal is explored, and found to be ambiguous. However, the state corresponding to a neutron crystal is identified, and the leading quantum correction to the classical mass is computed. The results are compared to those of Klebanov for a crystal of whole skyrmions.

1   Introduction

The Skyrme model [1] has had considerable qualitative success in describing both single nucleon properties and the nucleon-nucleon interaction [2, 3, 4]. This has prompted speculation as to whether the model might not also provide a reasonable description of dense nuclear matter, such as may exist in the interior of a neutron star. The idea of using a skyrmion crystal for this purpose was first raised by Klebanov [5]. There is some debate as to whether or not an ordered crystalline state is energetically preferable to a disordered neutron superfluid at high densities [6]. Klebanov considered a simple cubic array of skyrmions, appropriately rotated to ensure maximal attraction between each skyrmion and its six nearest neighbours. Subsequent investigations [7, 8] of this crystal revealed a phase transition: at high densities, the crystal becomes a bcc array of half-skyrmions (this was first realised by Manton and Goldhaber [9], who also identified an additional symmetry). The energy minimum occurs in this high-density phase. Different crystal symmetries were then tried [10]. The lowest energy configuration known consists of a simple cubic lattice of half-skyrmions [11, 12]. While it cannot rigorously be proved that this is indeed the lowest energy possible in the Skyrme model, it seems very likely that it is. The energy obtained is only 3.8% above the unreachable topological lower bound [1]. We therefore refer to it as "the" Skyrme crystal.

Before the Skyrme crystal can be used to describe nuclear matter in a neutron star, its global isospin must be quantised to ensure electrical neutrality. It is generally agreed that a proper treatment of the Skyrme model as a quantum field theory is extremely difficult. Instead, a semi-classical quantisation is usually performed [2], whereby the classical degrees of freedom of a given mode are treated as collective coordinates. thus reducing the model to a finite-dimensional quantum mechanics. The conventional wisdom is that $6N$ degrees of freedom are required to describe a system containing N nucleons (the same number as would be required to describe 6 widely separated skyrmions). For the Skyrme crystal, global isospin rotations should give the largest single quantum correction to the classical mass. The global rotation of an infinite crystal requires infinite energy; however a global isospin rotation requires only a finite amount of energy *per baryon*. Klebanov calculated the isorotational energy of his crystal, but this has so far been neglected for the minimum energy Skyrme crystal. There are some unresolved problems: skyrmion crystals predict too high a density for nuclear matter, and the remainder of the kinetic energy may be sufficient to unbind the crystal. It is therefore one of the aims of this letter to see how Klebanov's results are modified for the true Skyrme crystal. The classical Skyrme crystal is already known to have a higher binding energy and a lower density than Klebanov's crystal. We find that this tendency is even more heavily emphasised after the quantisation of global isospin, so that it would seem very unlikely that the Skyrme crystal could become unbound. Also, this (rather limited) quantisation already leads to a 25% correction to the classical density. The quantisation of the remainder of the $6N$ modes (which probably correspond to soft isospin and vibrational waves) is an extremely difficult problem, but the results of the present calculation indicate that tackling it may prove worthwhile.

We also desire to investigate the meaning of global isospin in an infinite crystal. Isospin is conventionally defined in the Skyrme model as an SO(3) rotation of the pion fields only. However, the full symmetry of the Skyrme Lagrangian is somewhat larger: chiral $SU(2)_L \times SU(2)_R \cong$ SO(4). This is usually broken for finite energy configurations by the necessity of setting the field to a constant value at spatial infinity (conventionally $U \to 1$ as $r \to \infty$). However, this condition does not apply to an infinite crystal. Also, it has been argued [13, 10] that the high density half-skyrmion phase of the Skyrme crystal corresponds to a restoration of chiral symmetry. In this case, there is no natural way to select the diagonal subgroup corresponding to isospin. This would not matter if all the fields transformed in the same way under the crystal point groups. Unfortunately, however, they do not: one field is singled out. The spectrum obtained by quantising global isospin is therefore dependent on whether or not this field is included in the diagonal subgroup. A unique energy spectrum can only be obtained by quantising the full chiral (SO(4)) symmetry. However, the interpration of isospin in these energy states is ambiguous, if indeed it can be meaningfully defined at all. This problem was not considered by Klebanov for

his crystal (though it should also arise), as his calculation had a particular definition of isospin built in from the start. The present letter therefore represents the first attempt to address these issues.

2 Calculations

In dimensionless units, the Skyrme Lagrangian density is [14]

$$\mathcal{L} = \frac{1}{2} Tr(L_\mu L^\mu) + \frac{1}{16} Tr([L_\mu, L_\nu][L^\mu, L^\nu]), \tag{1}$$

where $L_\mu = U^\dagger \partial_\mu U$ and $U = \sigma + i\underline{\pi}.\underline{\tau}$ is the SU(2)-valued scalar field (τ_i are the Pauli matrices). In these units the topological lower bound is $12\pi^2$ per baryon.

We will begin by considering standard $SO(3)$ isospin, and then generalise to $SO(4)$ at the most convenient point. A global isospin rotation of a Skyrme field U is defined

$$U(x) \mapsto AU(x)A^\dagger, \tag{2}$$

where A is an arbitrary $SU(2)$ matrix. This corresponds to an SO(3) rotation of the pion fields $\pi(x) \mapsto D(A)\pi(x)$, where $D(A)$ is the $SO(3)$ matrix associated to A via $D(A)_{ij} = \frac{1}{2}\text{Tr}(\tau_i A\tau_j A^\dagger)$. This transformation is now allowed to depend on time

$$U(\underline{x}, t) = A(t)U_0(\underline{x})A^\dagger(t), \tag{3}$$

giving rise to kinetic terms in the Lagrangian. The components of A are often treated directly as collective coordinates [2], but we will instead follow the procedure of [15] by defining the body-fixed angular velocity ω for iso-rotations to be

$$\underline{\omega}.\underline{t} = A^\dagger \dot{A}, \tag{4}$$

where $t_i = -\frac{1}{2}\tau_i$. The momentum conjugate to the angular velocity ω will then be the body-fixed angular momentum in iso-space.

Substituting (3) and (4) into the Lagrangian density (1), the kinetic energy of the Skyrme crystal is

$$T = \frac{1}{2} V_{ij} \omega_i \omega_j \tag{5}$$

where

$$
\begin{aligned}
V_{ij} &= \int d^3x \left\{ \frac{1}{4}\text{Tr}\left(U^\dagger[\tau_i, U]U^\dagger[\tau_j, U] \right) \right. \\
&\quad \left. + \frac{1}{16}\text{Tr}\left([U^\dagger[\tau_i, U], U^\dagger\partial_k U][U^\dagger[\tau_i, U], U^\dagger\partial_k U] \right) \right\}
\end{aligned}
\tag{6}
$$

is the isospin inertia tensor.

Equation (5) can now be generalised to SO(4) rotations of the fields

$$T = \frac{1}{2} V_{(ij)(kl)} \omega_{(ij)} \omega_{(kl)}, \tag{7}$$

where each of the indices runs from 0 to 3. Each of the pairs (ij) is antisymmetric under the interchange of the two indices, while the matrix V is symmetric with respect to its 'double' indices. We choose to label V by the pairs (01), (02), (03), (23), (31) and (12), in that order. Most elements of $V_{(ij)(kl)}$ can be calculated using the SO(3) formula (6). The few remaining elements are the cases where all four indices are different. They can be computed by considering the case of motion of constant (but different) velocity in two orthogonal planes, and selecting out the cross-terms.

The form of the inertia tensor V is strongly constrained by the symmetry of the Skyrme crystal. There are two kinds of point about which the Skyrme crystal has cubic symmetry: the centres of the half-skyrmions (where the baryon density is peaked), and the points where the 'corners' of the deformed half-skyrmions meet (where the baryon density is zero). The field transformations associated with either point group are sufficient to define the crystal. Kugler and Shtrikman [11] found the crystal fields by defining one point group, writing down the most general Fourier expansion consistent with this symmetry, and then determining the coefficients numerically by minimising the energy. In an independent study, Castillejo et al. [12] discovered that right at the energy minimum, the crystal fields are extremely well approximated by analytic formulae

$$
\begin{aligned}
\sigma &= \sin\alpha \sin\beta \sin\gamma \\
\pi_1 &= \cos\alpha \sqrt{1 - \frac{1}{2}\cos^2\beta - \frac{1}{2}\cos^2\gamma + \frac{1}{3}\cos^2\beta\cos^2\gamma}
\end{aligned}
\tag{8}
$$

and cyclically for π_2 and π_3. $\alpha = \frac{\pi x}{L}$, $\beta = \frac{\pi y}{L}$ and $\gamma = \frac{\pi z}{L}$, where L is the lattice parameter. These formulae are a three dimensional analogue of an exact two dimensional solution for the non linear σ model. Since these formulae encapsulate the symmetry of the crystal, which is the most important feature for our purposes, we adopt them for convenience. The form given in Equation (8) assumes the origin to be at the centre of the second point group mentioned above, but the symmetry with respect to the first point group can easily be found by applying the translation $x_i \mapsto x_i + L/2$ to these fields.

The full cubic point group consists of forty eight elements, which can be divided into ten equivalence classes, each corresponding to a particular physical symmetry of a cube. Associated to each element g of the cubic group, there is a linear transformation of the fields $\mathcal{D}(g)$, where $\mathcal{D}(g)$ is a 4×4 matrix. The matrices $\mathcal{D}(g)$ define a 4-dimensional representation of the group. Surprisingly, the field transformations associated with the two point groups of the Skyrme crystal correspond to different representations of the cubic group. The cubic group has

ten irreducible representations (irreps), four of which are 1-dimensional, two 2-dimensional and four 3-dimensional. The representation formed by the crystal symmetry (for either point group) is 4-dimensional, and must therefore be reducible. For the first point group (about centres of half-skyrmions), the representation can be decomposed into the trivial 1-dimensional irrep and a 3-dimensional irrep (the latter corresponding to the transformations of the Cartesian axes under the cubic group). For the second point group, the representation decomposes into two 1-dimensional irreps (one of which is the trivial representation) and a 2-dimensional irrep.

The crystal fields display a higher degree of symmetry with respect to the first point group than to the second. The first point group therefore imposes stronger constraints on the form of the inertia tensor (including the conditions imposed by the second). The decomposition into a 1-dimensional and a 3-dimensional irrep singles out one direction in isospace. There are thus two principal moments of inertia, one for iso-rotations purely within the 3-dimensional irrep (B), and one for iso-rotations which mix the two (A).

This analysis is confirmed by numerical computation. The principal moments of inertia depend on the lattice parameter L

$$
\begin{aligned}
A &= 6.6667L^3 + 69.4944L, \\
B &= 9.3333L^3 + 74.9876L.
\end{aligned}
\tag{9}
$$

All spatial integrations were performed over one unit cell. This is necessary to ensure that the full symmetry of the crystal is exhibited. The values for the principal moments of inertia given above should therefore be interpreted as being the moments *per unit cell*. A unit cell consists of eight half-skyrmions in a cube of side length $2L$, centred at the origin of one of the point groups.

The inertia tensor is diagonal if and only if a single field (rather than a linear combination of fields) is chosen to correspond to the trivial irrep. At this point, we note that the labelling of the inertia tensor $V_{(ij)(kl)}$ implies a choice of the diagonal subgroup (SO(3) isospin). It is clear that the diagonal elements of $V_{(ij)(kl)}$ are A (3 times) and B (also 3 times). However, the *order* will vary according to whether the field chosen to correspond to the trivial irrep is σ, or one of the pion fields. This will affect the form of the classical Hamiltonian when it is expressed in terms of left and right SU(2) operators. Since the Lagrangian is chirally invariant, the quantised energy spectrums should be the same. However, the interpretation of isospin may vary. We will consider two cases to illustrate this point, assigning σ and π_3 respectively to the trivial irrep. The resulting inertia tensors are

$$
V_1 = \left(\begin{array}{ccc|ccc}
A & 0 & 0 & 0 & 0 & 0 \\
0 & A & 0 & 0 & 0 & 0 \\
0 & 0 & A & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & B & 0 & 0 \\
0 & 0 & 0 & 0 & B & 0 \\
0 & 0 & 0 & 0 & 0 & B
\end{array}\right), \qquad
V_2 = \left(\begin{array}{ccc|ccc}
B & 0 & 0 & 0 & 0 & 0 \\
0 & B & 0 & 0 & 0 & 0 \\
0 & 0 & A & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & A & 0 & 0 \\
0 & 0 & 0 & 0 & A & 0 \\
0 & 0 & 0 & 0 & 0 & B
\end{array}\right),
\tag{10}
$$

where V_1 obviously corresponds to the case where σ transforms according to the trivial irrep.

For the first case ($V = V_1$), the Lagrangian can be written

$$L = \frac{1}{2}A\Omega^2 + \frac{1}{2}B\omega^2 - V. \tag{11}$$

Defining conjugate momenta

$$L_i = \frac{\partial L}{\partial \omega_i}, \qquad M_i = \frac{\partial L}{\partial \Omega_i}, \tag{12}$$

the Hamiltonian can be written

$$H = \frac{1}{2A}\mathbf{M}^2 + \frac{1}{2B}\mathbf{L}^2 + V. \tag{13}$$

At this point, we need to convert to operator form to follow the usual canonical quantisation procedure. However, M and L are not good quantum numbers, as $\mathbf{M}$ and $\mathbf{L}$ do not separately obey the correct angular momentum commutation relations. The symmetry algebra is

$$[L_i, L_j] = i\epsilon_{ijk}L_k, \qquad [M_i, M_j] = i\epsilon_{ijk}L_k, \qquad [L_i, M_j] = i\epsilon_{ijk}M_k. \tag{14}$$

This is the algebra of vector (L_i) and axial (M_i) $SU(2)$ transformations (as would be expected, given the labelling of the inertia tensor). Introducing

$$J_i = \frac{1}{2}(L_i + M_i), \qquad K_i = \frac{1}{2}(L_i - M_i), \tag{15}$$

we find that these operators *do* obey the correct angular momentum algebra

$$[J_i, J_j] = i\epsilon_{ijk}J_k, \qquad [K_i, K_j] = i\epsilon_{ijk}K_k, \qquad [J_i, K_j] = 0. \tag{16}$$

This is the chiral algebra of $SU(2)_L \times SU(2)_R$. Rewriting the Hamiltonian in terms of $\mathbf{J}$ and $\mathbf{K}$, we obtain

$$H = \left(\frac{1}{2A} + \frac{1}{2B}\right)\left(\mathbf{J}^2 + \mathbf{K}^2\right) + 2\left(\frac{1}{2B} - \frac{1}{2A}\right)\mathbf{J}.\mathbf{K} + V. \tag{17}$$

From (15), $\mathbf{J} + \mathbf{K} = \mathbf{L}$, so that

$$2\mathbf{J}.\mathbf{K} = \mathbf{L}^2 - \mathbf{J}^2 - \mathbf{K}^2. \tag{18}$$

Consider a crystal containing a large number n of unit cells. We assume strictly periodic boundary conditions and ignore edge effects (ie. we put the crystal in a periodic box). Following the usual canonical quantisation procedure, we convert to operator form

$$\begin{aligned} E^{tot} &= nM_{cl} + \frac{\hbar^2}{nA}\left(J^{tot}(J^{tot} + 1) + K^{tot}(K^{tot} + 1)\right) \\ &\quad + \hbar^2\left(\frac{1}{2nB} - \frac{1}{2nA}\right)L^{tot}(L^{tot} + 1). \end{aligned} \tag{19}$$

where M_{cl} is the classical mass of a unit cell.

We now consider the case where π_3 corresponds to the trivial irrep, so that the inertia tensor is V_2 of Equation (10). The Lagrangian is then

$$L = \frac{1}{2}B(\Omega_1^2 + \Omega_2^2) + \frac{1}{2}A\Omega_3^2 + \frac{1}{2}A(\omega_1^2 + \omega_2^2) + \frac{1}{2}B\omega_3^2 - V. \tag{20}$$

Using the same definitions of $\mathbf{L}$, $\mathbf{M}$, $\mathbf{J}$ and $\mathbf{K}$, the classical Hamiltonian is

$$\begin{aligned}
H &= \frac{1}{2B}(M_1^2 + M_2^2) + \frac{1}{2A}M_3^2 + \frac{1}{2A}(L_1^2 + L_2^2) + \frac{1}{2A}L_3^2 + V \tag{21}\\
&= \frac{1}{B}(\mathbf{J}^2 + \mathbf{K}^2) + \left(\frac{1}{2A} - \frac{1}{2B}\right)\mathbf{L}^2 + \left(\frac{1}{A} - \frac{1}{B}\right)(J_3^2 + K_3^2 - L_3^2) + V,
\end{aligned}$$

which on conversion to operator form (again for n unit cells) gives

$$\begin{aligned}
E^{tot} &= nM_{cl} + \frac{\hbar^2}{nB}\left(J^{tot}(J^{tot}+1) + K^{tot}(K^{tot}+1)\right) \\
&\quad + \hbar^2\left(\frac{1}{2nA} - \frac{1}{2nB}\right)L^{tot}(L^{tot}+1) \\
&\quad + \hbar^2\left(\frac{1}{nA} - \frac{1}{nB}\right)(J_3^{tot\,2} + K_3^{tot\,2} - L_3^{tot\,2}). \tag{22}
\end{aligned}$$

At first sight, this spectrum appears totally different to that given by Equation (19), but in fact the energy eigenvalues (and their degeneracies) are exactly the same.

The different forms (17) and (21) of the classical Hamiltonian reflect the differing choices of diagonal subgroup (SO(3) isospin). We can reduce the chiral Hamiltonians to SO(3) isospin Hamiltonians by setting $\mathbf{J} = \mathbf{K}$. Equation (17) then reduces to

$$H = \frac{1}{2B}\mathbf{L}^2 + V, \tag{23}$$

which gives the energy spectrum

$$E^{tot} = nM_{cl} + \frac{\hbar^2}{2nB}L^{tot}(L^{tot}+1). \tag{24}$$

Equation (21) reduces to

$$H = \frac{1}{2A}\mathbf{L}^2 + \left(\frac{1}{2B} - \frac{1}{2A}\right)L_3^2 + V, \tag{25}$$

with the corresponding energy spectrum

$$E^{tot} = nM_{cl} + \frac{\hbar^2}{2nA}L^{tot}(L^{tot}+1) + \frac{\hbar^2}{2}\left(\frac{1}{nB} - \frac{1}{nA}\right)(L_3^{tot})^2. \tag{26}$$

These spectrums *are* genuinely different. However, they give the same energy for an infinite neutron crystal. For a neutron crystal, $L^{tot} = 2n$, $L_3^{tot} = -2n$, remembering that there are four baryons per unit cell. Letting $n \longrightarrow \infty$, the energy per baryon (from either spectrum) in a neutron crystal is

$$\frac{E}{B} = \frac{1}{4}M_{cl} + \frac{\hbar^2}{2B}. \tag{27}$$

Note that the cancellation of all terms involving the moment A in the spectrum (26) only occurs in this limit. Effectively, the isospin becomes large enough to be treated classically, and the motion is reduced to rotation about one isospin axis.

3   Results and Conclusions

To summarise, the energy spectrum obtained from the quantisation of chiral symmetry is unique, but the interpretation of isospin in these energy states is ambiguous. If one only considered the spectrum (19), it would seem that all SO(4) energy eigenstates were of definite isospin (L^{tot}), degenerate with respect to L_3^{tot}. However, states of the same energy can be seen to have different isospin in spectrum (22). Worse, not all the energy eigenstates in this second case are also eigenstates of isospin. The last term of the Hamiltonian (21) mixes states of different isospin but the same third component (for given J^{tot} and K^{tot}).

These results are not altogether surprising. Since the Lagrangian is chirally invariant, it would be a shock to discover that the quantised energy spectrum was not. However, the chiral symmetry is slightly broken by the cubic symmetry of the Skyrme crystal, in that the representation formed by the fields is reducible. This singles out one field as 'special'. All the trouble with the interpretation of isospin stems from the presence of 'spin-orbit' type terms in the chiral Hamiltonians. These would disappear if all the principal moments of inertia were equal, as they would be if all the fields transformed in the same way under the cubic point group of the crystal.

One possible way to resolve this ambiguity is to consider a finite piece of crystal. It could be argued that for any physical application the crystal would indeed be finite, even if very large. A choice of vacuum at spatial infinity would then have to be made, breaking chiral symmetry in the usual way. Furthermore, to maintain compatibility with the cubic symmetry of the crystal, σ must be chosen to transform as the trivial irrep of whichever point group is centred at the origin. Unfortunately, the same field does not correspond to the trivial irrep for the two different point groups of the crystal, so that the diagonal subgroup would still depend on the choice of origin. The argument is further weakened by the idea that the high density half-skyrmion phase of the Skyrme crystal may correspond to a restoration of chiral symmetry [10, 13], which suggests that the crystal interior might not be affected by boundary conditions at its edge.

Despite these difficulties, the energy of a neutron crystal would appear to be uniquely defined by Equation (27). We will assume this to be true. The total, classical and isorotational energies per unit cell are shown in Figure 1. Part of this graph is also displayed on a larger scale, to highlight the difference in the value and position of the energy minimum, for the classical and quantised Skyrme crystal. The quantisation of global isospin has the effect of raising the minimum energy slightly from 491.81 to 513.84 (per unit cell) in our units: a difference of approximately 4%. If the parameters of the Skyrme model are chosen to fit the masses of the nucleon and the delta resonance, for zero pion mass [2], then our energy units are equivalent to 5.92MeV, our length unit equals 0.561 fm, and $\hbar = 59.4$ in our units. So, translated into real units, the difference in energy is $32\ MeV$. Perhaps more significant is the fact that the value of the lattice parameter at the minimum increases from 2.35 to 2.54 in our units. This means
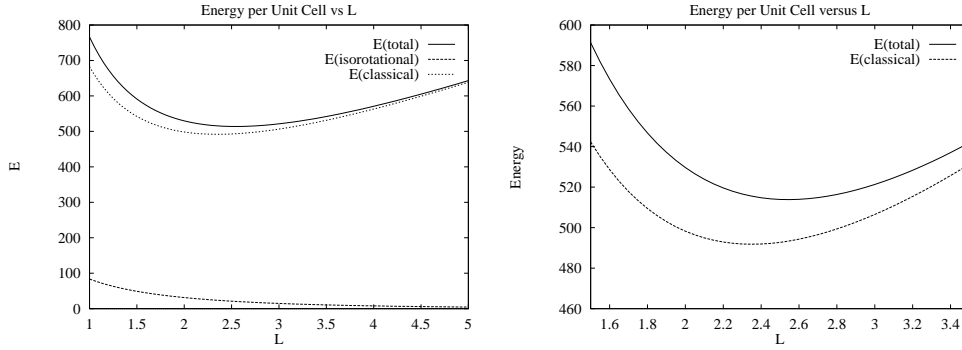
Figure 1: Classical and isorotational energies per unit cell, and their sum, versus the lattice parameter L.

that the quantised crystal is almost 25% less dense (0.173 baryons/fm^3 as compared to 0.218 baryons/fm^3) than the classical Skyrme crystal.

To facilitate comparison with the results of Klebanov [5] for the crystal of whole skyrmions, we have also plotted the total energy per baryon against the volume per baryon in standard units (see Figure 2). The true Skyrme crystal has a higher binding energy, as would be expected. More surprisingly, it is also less dense than the crystal considered by Klebanov. As mentioned earlier, there is general agreement that the Skyrme crystal predicts too high a density for nuclear matter. This calculation seems promising in this respect, in that a crystal with lower energy *and* density than that considered by Klebanov has been found. The crystal is still too dense, but it must be remembered that the quantisation carried out is only partial: only 6 out of the $6N$ modes which should be included were actually considered (global isospin, plus the zero momentum vibrational modes). This is vanishingly few per baryon. It seems likely that quantisation of the remaining modes will lead to further corrections to the density of the crystal. Allowing vibrational modes will almost certainly tend to make the lattice parameter increase, and additional isospin modes should also have the same trend, by analogy with global isospin.

It is difficult to know how much of the total kinetic energy per baryon has been included in the present calculation. Klebanov suggests that the total kinetic energy per baryon should be of order 100 MeV for his crystal, which could be enough to unbind it. It is equally impossible to be sure that the true Skyrme crystal will be bound, once the proper zero point energy is included. However, the Skyrme crystal seems much more likely to be bound, since the binding energy of the partially quantised neutron crystal is approximately 20% higher than for the Klebanov crystal. Overall, the trends obtained from the limited quantisation performed here are extremely promising. A full quantisation of the remaining modes, although difficult, would appear to be interesting and worthwhile in the light of these results.
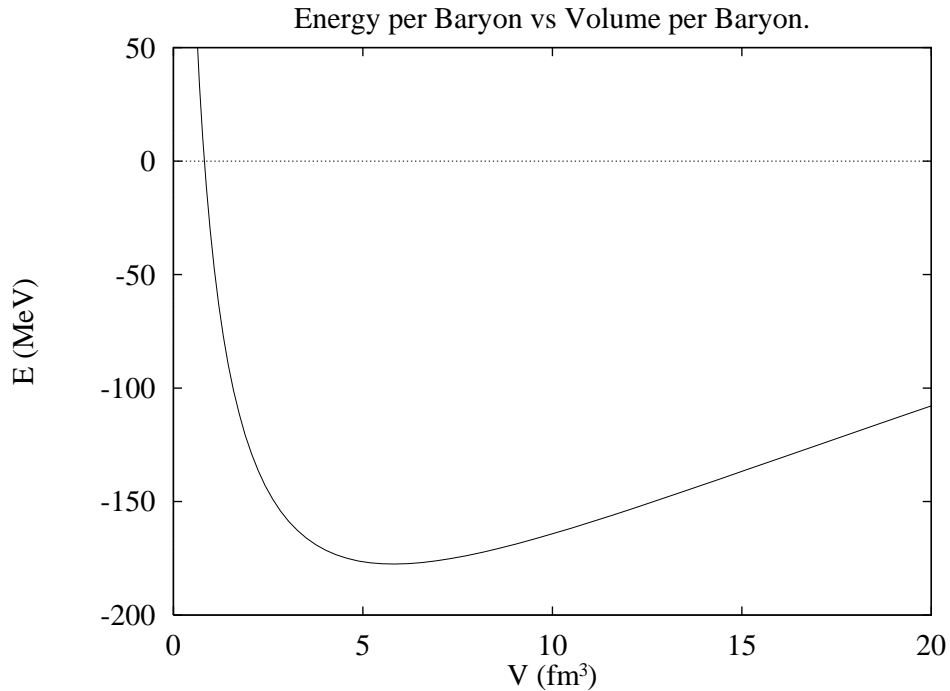
Figure 2: Sum of classical and isorotational energies per baryon versus volume per baryon. The nucleon mass (938 MeV) has been subtracted.

Acknowledgements

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